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Deriving Gaussian Fatigue Test Spectra from Measured non Gaussian Service Spectra

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Abstract

A well-known problem is the transition from measured random vibration service loads to synthetically generated test signals. They are used for experimental vibration tests modelling the fatigue potential of long-lasting real service loads within a limited test interval. The challenge of this transition is caused by the non-Gaussian character of the service loads and the fact that the fatigue loads that have to be kept for the tests are caused by the response vibrations of an arbitrary structure. The analysis presented here is based on the finding that the non-Gaussian character of the service load is a function of frequency.

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Keywords: non-Gaussian random vibrations; power spectral density (PSD); fatigue strength; accelerated vibration testing; fatigue damage spectrum; railway engineering;

1. Introduction

Fatigue problems in mechanical systems are often related to random vibrations. Methods for the analysis of the impact of these vibrations on the fatigue strength of dynamical structures are a subject of ongoing research activities. Especially in the area of non-Gaussian vibrations engineers still have to accept a lack of reliable tools. These tools are related to theoretical techniques that are applied on fatigue strength calculations as well as to experimental methods used for fatigue testing. The analysis presented here is related to the generation of test spectra used for experimental

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fatigue strength tests for dynamical systems. Usually these tests are based on an accelerated testing which means that a measured random service load signal (e.g. an acceleration signal of a railway bogie) of a limited duration (e.g. some hours) is taken as a sample representing the total lifetime of the system (in railway engineering this total lifetime may have values of up to 30 y); then this sample signal is manipulated in a way that it causes the same fatigue load on the tested component in a limited test duration (e.g. 200 h). As this test signal acts as an excitation of a vibration system (e.g. some bogie mounted equipment), it cannot simply be replicated faster because this would change its spectral properties and consequently the response of the excited vibration system. A typical procedure to solve this problem is to derive the power spectral density (PSD) from the measured signal and then to use this PSD as a test signal together with an amplification factor, which is derived from the acceleration factor, the slope of the S-N curve and Miners rule. Usual test equipment is able to derive Gaussian distributed time signals from this test PSD. Consequently this described procedure is based on the assumption of a Gaussian distribution of the measured time signal.

In real life random signals are often not exactly Gaussian but as long as the deviation from the theoretical Gaussian assumption is small, the error in the tested fatigue life is acceptable. Especially in railway engineering this assumption often causes relevant deviations in the fatigue life that must not be neglected. Therefore numerous procedures and improvements have been proposed (e.g. [1, 2, 3, 4, 5]) dealing with the impact of non-Gaussian distributions on fatigue load in different ways. The material presented in this paper is based on the finding that the deviation of the probability distribution of a measured signal from the theoretical Gaussian distribution is not just one single characteristic for the entire frequency range of the signal. The presented results show that this deviation depends on the frequency of the signal. A filtering of non-Gaussian time signals for limited frequency bands usually shows a non-constant deviation from the Gaussian distribution over the frequency spectrum of these time signals. Consequently the difference between the fatigue load of a non-Gaussian signal and its corresponding Gaussian signal is also depending on the signal's frequency. Based on this fact, a procedure for the derivation of test spectra is developed. For the presentation of the material in the following sections it is assumed that readers are familiar with basic concepts of random vibrations (Fourier-series, frequency response function, probability distribution, power spectral density, see e.g. [6]) and fatigue damage calculations (e.g. Rainflow counting, Palmgren-Miner rule, S-N curve).

2. Analysis of a measured non-Gaussian vibration signal

Fig. 1(a) shows the acceleration $a_M(\mathrm{t})$ measured at the bogie frame of a rail vehicle. The measurement has a duration of $T_M = 1250$ s and a standard deviation of $\sigma = 1.12\ \frac{\mathrm{m}}{\mathrm{s}^2}$. Based on the assumption that this acceleration is repeated for a total service life time of $T_{tot} = 27 \cdot 10^7$ s its equivalent fatigue load $a_{M,equ}$ may be calculated by applying Rainflow Counting (mean stresses neglected), a S–N–diagram and Miners rule

$$a_{equ}^k N_{equ} = \sum_i a_i^k N_i \tag{1}$$

where k is the exponent of the S–N–diagram, N_{equ} is the number of reference load cycles for the equivalent load a_{equ}, a_i is the load of the i–th load class and N_i is the number of load cycles of the i–th load class (here $k = 3$ and $N_{equ} = 10^7$ are used as typical values for steel materials, for a comparison also $k = 5$ is used later).

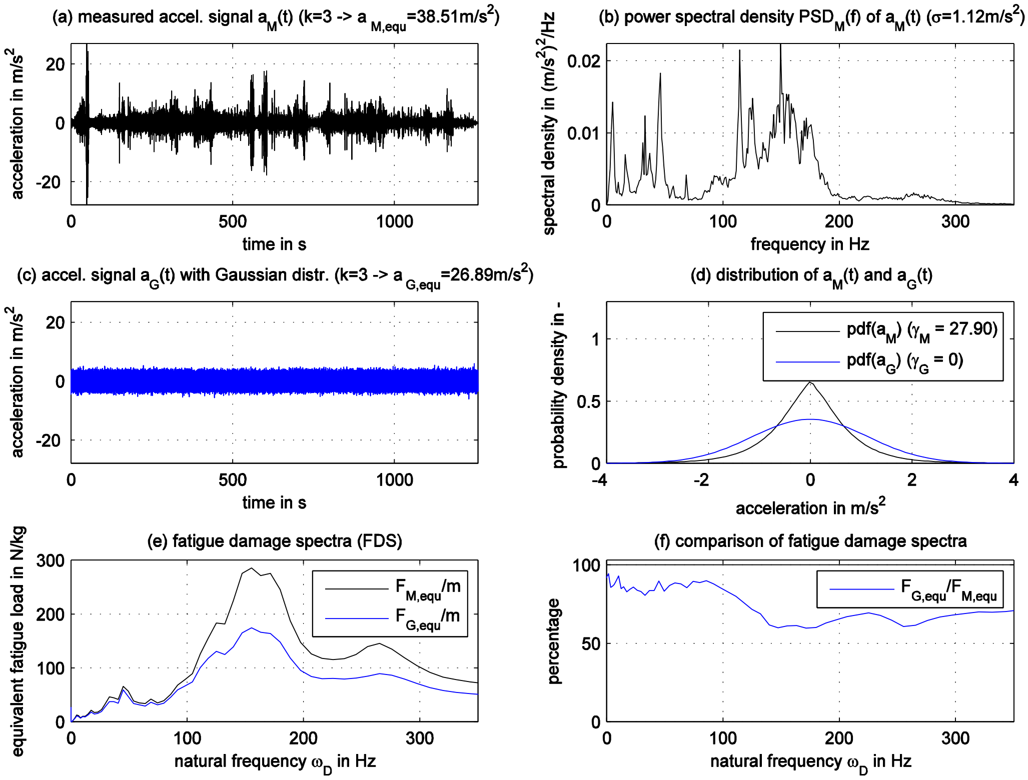

Fig. 1. (a) measured acceleration signal $a_M(\mathrm{t})$; (b) power spectral density $PSD_M(\mathrm{f})$; (c) Gaussian acceleration signal $a_G(\mathrm{t})$; (d) probability distributions; (e) fatigue damage spectra for k=3; (f) comparison of fatigue damage spectra.

The resulting value of $a_{M,equ} = 38.51\ \frac{\mathrm{m}}{\mathrm{s}^2}$ expresses the damage potential of the acceleration signal $a_M(t)$ with one single value; it means that a load cycle of that magnitude repeated for $N_{equ} = 10^7$ times, causes the same fatigue damage as the real random service load. The corresponding power spectral density $PSD_M(f)$ is depicted in Fig. 1(b); it shows that $a_M(t)$ is a broad band vibration signal. By assuming a Gaussian distribution, this PSD may be used to derive a time signal $a_G(t)$ having the same power spectrum (and the same standard deviation) as the measured signal $a_M(\mathrm{t})$; see Fig. 1(c). Fig. 1(d) shows the real probability distribution of $a_M(t)$ compared to a Gaussian distribution. The comparison of Fig. 1(a) and Fig. 1(c) seems to show short time overloads in $a_M(\mathrm{t})$ that do not appear in the synthetically generated signal $a_G(t)$. This difference may be understood not just as a non-Gaussian versus a Gaussian signal but also as a non-stationary $a_M(\mathrm{t})$ versus a stationary signal $a_G(t)$. As a clear differentiation between non-Gaussian and non-stationary is a big issue the signal $a_M(\mathrm{t})$ was interpreted just as non-Gaussian here; this is clearly confirmed by the distributions shown in Fig. 1(d) ([7] presents an analysis of the correlation between non-Gaussian and non-stationary random signals).

To quantify the non-Gaussian character of the distribution of $a_M(t)$, the kurtosis γ may be applied. Where $\gamma = 0$ corresponds to a Gaussian distribution, $\gamma > 0$ corresponds to a leptokurtic (densities for very small values close to the mean $\bar{a}$ and for large values far away from the mean are higher compared to Gaussian), $\gamma < 0$ to a platykurtic distribution (densities for very small values close to the mean $\bar{a}$ and for large values far away from the mean are smaller compared to Gaussian). Eq. (2) summarizes the relevant formulas based on a signal $a_M(t)$ given in a digitally sampled form $a_M(t_r) = a_r$ for $r = 1 \ldots R$ and $t_r = r\,\Delta t$ (results for $a_M(\mathrm{t})$: $\gamma_M = 27.9$ / $a_G(\mathrm{t})$: $\gamma_G = 0$).

$$\bar{a} = \frac{1}{R}\sum_{r=1}^{R} a_r\ ; \quad m_i = \frac{1}{R}\sum_{r=1}^{R}(a_r - \bar{a})^i\ ; \quad \sigma^2 = m_2\ ; \quad \gamma = \frac{m_4}{(m_2)^2} - 3\ ; \tag{2}$$

Equivalently to the fatigue load $a_{M,equ}$ of the measured signal, also the fatigue load $a_{G,equ}$ of the synthetically generated Gaussian distributed signal may be calculated. The resulting fatigue load of $a_{G,equ} = 26.89\ \frac{\mathrm{m}}{\mathrm{s}^2}$ differs from $a_{M,equ}$; the deviation is about 30 % (for $k = 5$ the fatigue loads are $a_{M,equ} = 23.86\ \frac{\mathrm{m}}{\mathrm{s}^2}$ and $a_{G,equ} = 10.29\ \frac{\mathrm{m}}{\mathrm{s}^2}$ with an even higher deviation of about 57 %). These results show clearly that the probability distribution of the measured excitation signal has an impact on the equivalent acceleration. A reduction of this signal to just its PSD (which is equivalent to the assumption of a Gaussian distribution) may cause a relevant deviation from the real fatigue load.

Finally not the equivalent fatigue load a_{equ} of the excitation signal is relevant for the fatigue damage of a certain structure; it is the stress response of the vibration system that matters. But this stress response is unknown because the dynamical behavior of the vibration system is unknown as well. Usually a signal derived for fatigue tests is supposed to serve experiments with different test objects having different vibration properties. As a consequence, the fatigue potential of an excitation signal can only be assessed properly by analyzing the stress responses it produces under the assumption of an arbitrary vibration system with variable parameters. This concept is called fatigue damage spectrum (FDS) [4] and can be based on the assumption of a linear single degree of freedom oscillator (Fig. 2) with the excitation $a(t)$ and a variable natural frequency ω_D (displacement *y(t)*, stiffness *c*, damping *d*, mass *m*). The differential equation and the relevant parameters are given in Eq. (2).

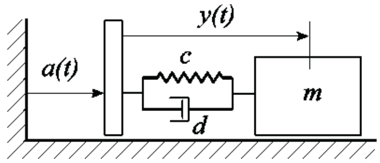

Fig. 2. single degree of freedom oscillator with an excitation acceleration *a(t)* and the response displacement *y(t)* .

$$m\,\ddot{y}(t) + d\,\dot{y}(t) + c\,y(t) = -m\,\ddot{u}(t)\ ;$$

$$\ddot{y}(t) + 2D\omega_o\,\dot{y}(t) + \omega_o^2\,y(t) = -\ddot{u}(t)\ ; \qquad \text{with: } \omega_o = \sqrt{\frac{c}{m}}\ ;\ D = \sqrt{\frac{d}{2\,m\,\omega_o}}\ ,\ \omega_D = \omega_o\sqrt{1-D^2} \tag{3}$$

Due to the unknown dimensions, the real stress response is unknown; it may be replaced by the sum of forces in the spring-damper-element $F(t) = cy(t) + d\dot{y}(t)$. Therefore a frequency response function may be derived based on a complex approach (with the imaginary number *j*, the excitation frequency *f*, the excitation amplitude $\hat{a}$ and the response amplitude $\hat{F}$):

$$\frac{\left(\hat{F}/m\right)}{\hat{a}}(j2\pi f) = -\frac{\omega_o^2 + 2D\omega_o\,j2\pi f}{\omega_o^2 + 2D\omega_o\,j2\pi f - (2\pi f)^2} \tag{4}$$

It turns out that the only input magnitudes needed to analyze this frequency response function are the natural frequency ω_o of the oscillator, its damping (here a typical damping factor D = 3 % is used) and the frequency f of the excitation signal (which will be replaced by the total frequency interval of $a(t)$). The response amplitude has the dimension of force-amplitude $\hat{F}$ per mass m. Consequently there is no need to define a concrete mass m of the oscillator because the results may be expressed in a normalized form which is sufficient for the comparison of different results. By performing a variation of the natural frequency ω_o of the oscillator, the possible vibration responses of an unknown arbitrary system may be analyzed. In addition, the equivalent fatigue load of these responses may be calculated and plotted for different values of ω_o. This plot is called fatigue damage spectrum and it is presented in Fig. 1(e) (here the damped natural frequency ω_D is used in the abscissa of the plot, see Eq. (2)) for the measured signal $a_M(t)$ and the synthetically generated signal $a_G(t)$. Consequently the corresponding fatigue damage results are called $\frac{F_{M,equ}}{m}(\omega_D)$ and $\frac{F_{G,equ}}{m}(\omega_D)$ (in addition Fig. 1(f) shows a comparison of these results in %). It is obvious that these two curves show a difference which is caused by the switch from the real to a Gaussian probability distribution. In addition, it has to be observed that this difference is not constant in the fatigue damage spectrum but it is depending on the natural frequency ω_D of the oscillator. A rough guess might have been to say that this difference is caused by the relation of $a_{M,equ}$ to $a_{G,equ}$, but this attempt to explain the deviation seems to not be sufficient.

3. Generation of an accelerated test PSD

To resolve the problem seen in chapter 2 and to derive an accelerated test signal $PSD_{test}(f)$, an amplification factor b for the Gaussian distributed signal $a_G(t)$ may be defined in the following way (with the test duration T_{test} and the total service life time of T_{tot})

$$b = \underbrace{\left(\frac{T_{tot}}{T_{test}}\right)^{\frac{1}{k}}}_{b_T} \cdot \underbrace{\left(\frac{a_{M,equ}}{a_{\mathrm{G},equ}}\right)}_{b_G} \tag{5}$$

where the amplification factor b_T covers the reduced time and b_G covers the transition from a non-Gaussian to a Gaussian distribution. For example, for a test duration of $T_{test} = 25\ \mathrm{h}$ and $k = 3$ the result is $b = 60.4$; the test signal is $a_{test}(t) = b\, a_G(t)$ (for the power spectral densities this factor has to be squared: $PSD_{test}(f) = \mathrm{b}^2\, PSD_M(f)$). If the fatigue damage spectrum for this test signal $PSD_{test}(f)$ is calculated the same way as it was done in chapter 2 for the service signal $a_M(t)$ and its Gaussian counterpart $a_G(t)$, the impact of the acceleration factor b on the fatigue load of the test object may be assessed. These corresponding results are summarized in Fig. 3. Basically Fig. 3(a) and Fig. 3(b) are similar to Fig. 1(e) and Fig. 1(f), just the results for the equivalent fatigue load $F_{test,equ}(\omega_D)$ for the test signal are added; Fig. 3(c) and Fig. 3(d) show the corresponding results for $k = 5$.

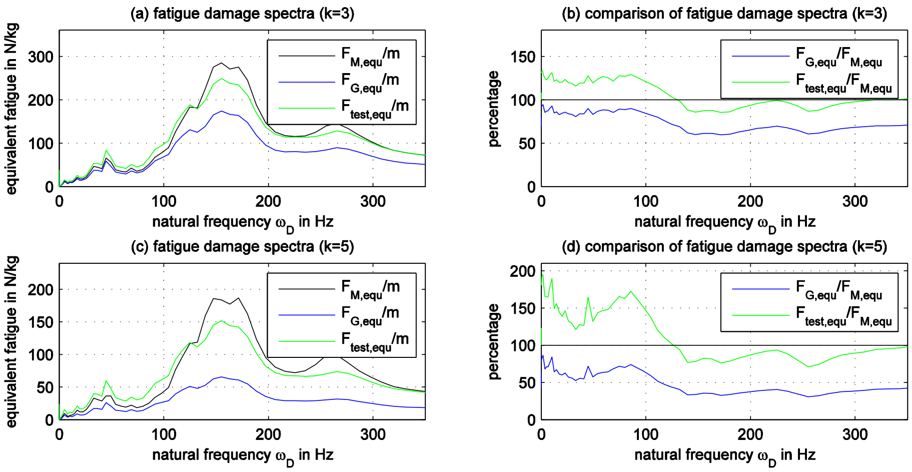

Fig. 3. (a) fatigue damage spectra for k=3; (b) comparison of fatigue damage spectra; (c) fatigue damage spectra for k=5; (d) comparison of fatigue damage spectra.

The consequence is obvious and simple to summarize: test objects having natural frequencies in the interval of about 0 to 100 Hz will be tested too hard; test objects with natural frequencies higher than 125 Hz will not be tested hard enough. And the impact of a growing k seems to make the situation even worse!

4. Improved generation of an accelerated test PSD

As already introduced in chapter 1, it was found that real non-Gaussian signals tend to have a non constant probability distribution in the frequency domain. In other words: the deviation from the Gaussian distribution depends on the frequency of the signal and consequently the factor b_G used to correct this deviation in terms of fatigue loads is not any more just one value for the total frequency interval, but it has to respect a variable deviation depending on the frequency of the signal. In order to introduce an improved method for the generation of Gaussian test signals, a signal filtering is used. By applying the following Fourier series, the signal $a_M(t)$ may be transformed into the frequency domain:

$$\mathrm{a}_M(t) = \frac{C_o}{2} + \sum_{l=1}^{L} C_l \cos(\underbrace{l\ \Delta f\ 2\pi}_{\omega_l}\ t\ +\ \phi_l)\ ;\quad \text{with:}\quad l = 1 \ldots L \tag{6}$$

This equation shows the correspondence between $a_M(t)$ and a digital spectrum with L data points, in which the smallest frequency $\Delta f = \frac{1}{T_M}$ is defined by the duration T_M of the measurement $a_M(\mathrm{t})$. The amplitudes C_l and the phase angles ϕ_l of the corresponding frequency $f_l = l\ \Delta f$ (angular frequency $\omega_l = l\ \Delta f\ 2\pi$) follow from the solutions of the Fourier integrals [6]. To obtain information on the probability distribution depending on the frequency f of the signal $a_M(\mathrm{t})$, a filtering of the above introduced Fourier spectrum (C_l ; ϕ_l) for defined frequency intervals n (with: $n = 1 \ldots N$) is used. As illustrated in Fig. 4 filtered signals $a_{M,n}(\mathrm{t})$ are derived from only the amplitudes C_l and phase angles ϕ_l located within the frequency interval n to be analyzed.

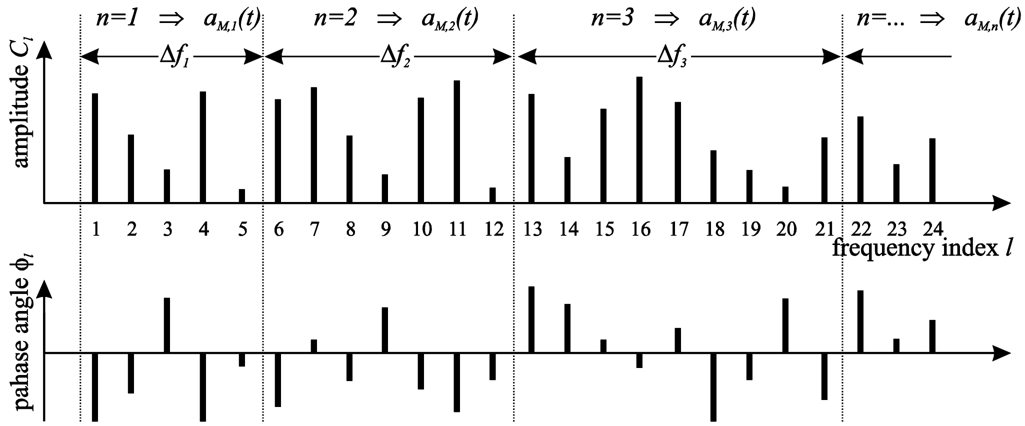

Fig. 4. Filtering of a_M(t) (respectively its Fourier spectrum C_l and ϕ_l) in intervals n.

Consequently N filtered signals $a_{M,n}(t)$ may be derived. In order to get information about the frequency distribution of the deviation from a Gaussian signal, the kurtosis values $\gamma_{M,n}$ of all these N filtered signals $a_{M,n}(t)$ may now be calculated. $a_M(t)$ was analyzed in $N = 30$ intervals of 12 Hz; Fig. 5 shows the corresponding results together with the kurtosis γ_M of the originally measured signal $a_M(t)$. As the kurtosis may be understood as a measure for the deviation from a Gaussian distribution, this result clearly highlights the fact that this deviation is not constant for the entire frequency spectrum of the signal. It has a distribution depending on the signal frequency. Consequently an accurate fatigue analysis of random vibrations requests to respect this frequency dependent and non constant deviation from the Gaussian distribution.

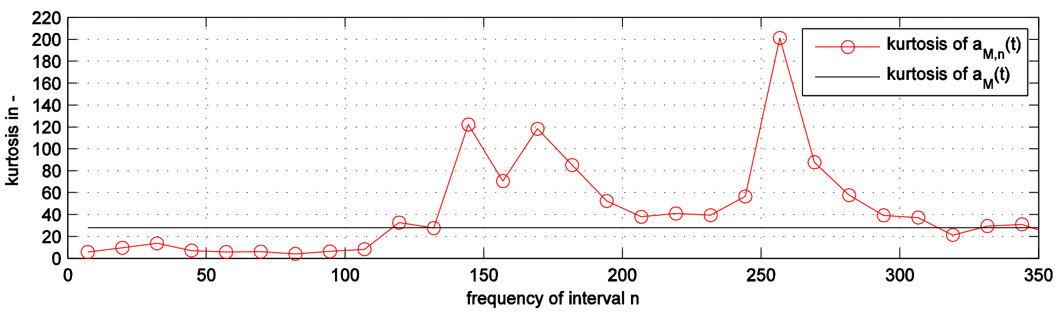

Fig. 5. kurtosis $\gamma_{M,n}$ of N filtered signals $a_{M,n}(t)$.

The basic idea of an improved procedure for the generation of test PSDs is to apply the method explained in chapter 3 not just for the total measured signal $a_M(t)$ (with one amplification factor b) but to use this concept for each single filtered signal $a_{M,n}(t)$ (each with its own specific amplification factor b_n , see Eq. (5)). As the portion b_T of the amplification factor b is just related to the time acceleration of the test, it will be identical for all the frequency intervals n; but the part b_G will be specific for each interval n because of different relations of the equivalent loads of the filtered measured signals $a_{M,n,equ}$ and its Gaussian counterpart $a_{G,n,equ}$.

$$b_n = \underbrace{\left(\frac{T_{tot}}{T_{test}}\right)^{\frac{1}{k}}}_{b_T} \cdot \underbrace{\left(\frac{a_{M,n,equ}}{a_{G,n,equ}}\right)}_{b_{G,n}} \tag{7}$$

Finally by summarizing these N signals $a_{imp,n} = b_n a_{G,n}(t)$ (or as PSD intervals: $PSD_{imp,n}(f) = b_n^2\, PSD_{M,n}(f)$), a new improved test signal $a_{imp}(t) = \sum_{n=1}^{N} a_{imp,n}(t)$ (or also $PSD_{imp}(f)$) may be derived for the total frequency range. This new signal is supposed to show smaller deviations in the fatigue damage spectrum, because the frequency dependent fatigue load gap between the real and the Gaussian distributed signals is corrected. After applying this concept to numerous examples, the predicted improvements were visible; but it also turned out that the equivalent fatigue load $a_{imp,equ}$ of the generated signals $a_{imp}(t)$ was not always exactly identical with $a_{M,equ}$. Therefore another correction factor b^* was introduced to adjust the correct fatigue load of the excitation signal. The final corrected excitation signal $a^*_{imp}(t)$ is derived as follows:

$$a^*_{imp}(t) = b^* a_{imp}(t)\,; \qquad \text{with:} \quad b^* = \frac{a_{M,equ}}{a_{imp,equ}}\,; \tag{8}$$

Fig. 6 summarizes the results achieved with this improved method in comparison to the results from Fig. 3. They were calculated with logarithmically growing widths of the N frequency bands. Basically Fig. 6(a) to (d) are similar to Fig. 3(a) to (d), just the new results for the equivalent fatigue loads $F^*_{imp,equ}(\omega_D)$ calculated based on $a^*_{imp}(t)$ are added. The improvement in the equivalent fatigue load is clearly visible. Especially the strong deviation in $F_{test,equ}(\omega_D)$ in the frequency interval from about 0 to 100 Hz could be reduced to a small acceptable deviation. Similar results have been achieved with different measurements form the area of railway bogies.

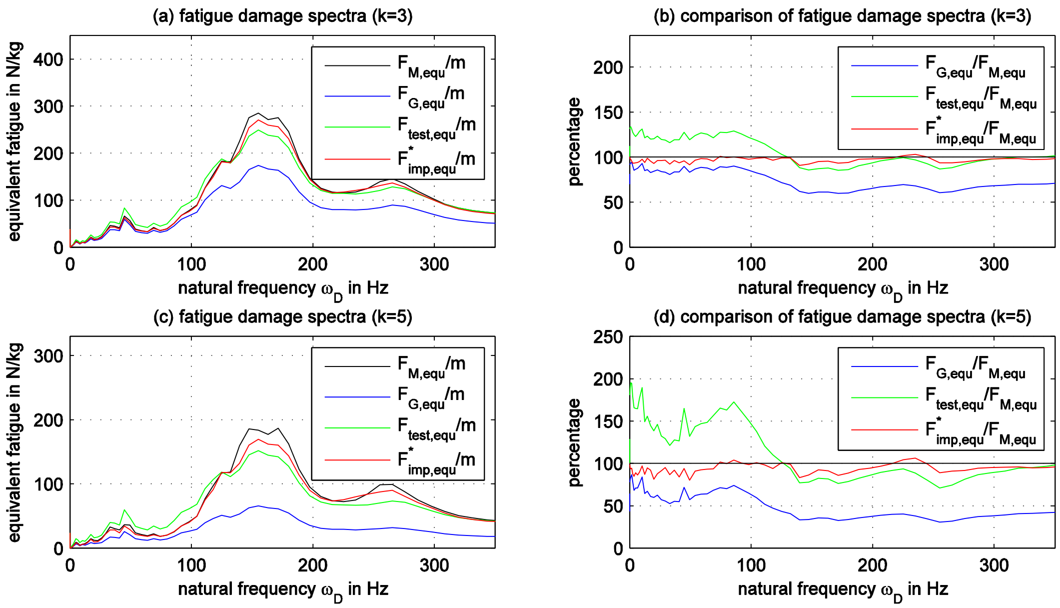

Fig. 6. (a) fatigue damage spectra for k=3; (b) comparison of fatigue damage spectra; (c) fatigue damage spectra for k=5; (d) comparison of fatigue damage spectra.

5. Conclusion

The presented work introduces a method for the generation of Gaussian fatigue test signals based on measured non-Gaussian service loads. It uses a frequency filtering to derive the correct frequency distribution of the fatigue load of the measured signal. The generated test signals may be applied for accelerated fatigue testing of dynamical structures. By using the concept of fatigue damage spectrum it was shown that the fatigue load of the stress response of an arbitrary dynamical system is modelled correctly. The proposed method was tested with different vibration signals measured in railway bogies.

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